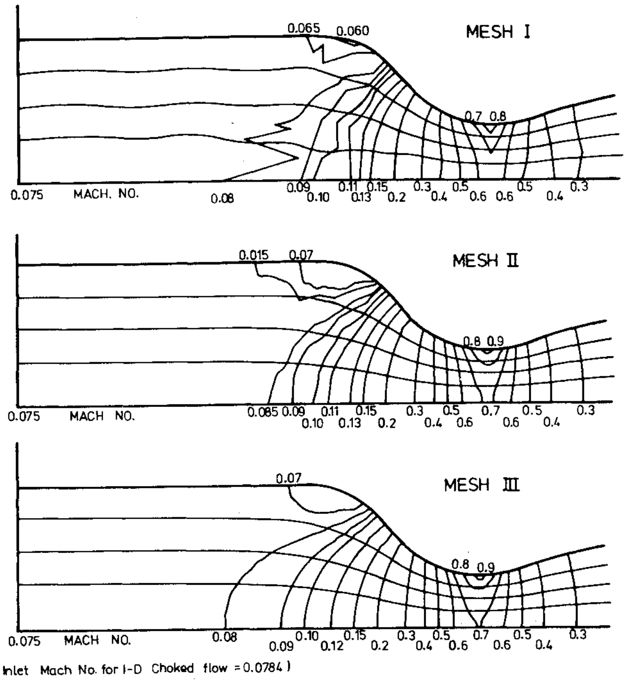

Fig. 2a Predicted compressible flowfield: −96% of one-dimensional mass flow.

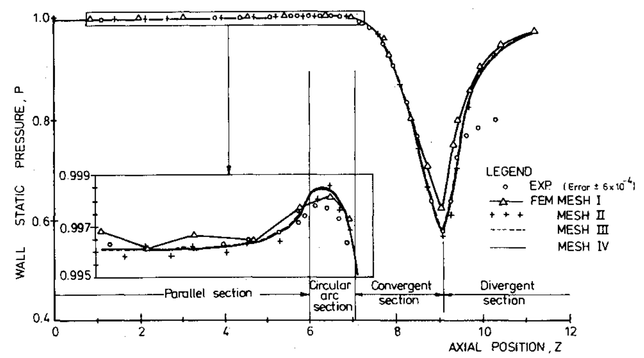

Fig. 2b Wall static pressure in compressible flow: −96% of one-dimensional choking mass flow.

Results and Discussion

Figure 2 gives computed results for the nozzle defined in Fig. 1. Four successively refined triangular element meshes were generated. Meshes II, III, and IV have element sizes 1/2, 1/4, and 1/6 of those of mesh I. The elementary one-dimensional solution value of ρ was used to start the iteration process. Linear interpolation functions were used to approximate ρ, u, and w.

The choked mass flow rate for this nozzle ($R_* = 2.0$) is ~99.4% of the one-dimensional choking mass flow.[10] Two cases of near choking flow are reported: 1) 96% of the one-dimensional choking mass flow and 2) flow with a small supersonic pocket, 99.3% of one-dimensional choking flow, the highest achieved. Table 1 gives computing times.

Experiments on the nozzle of Fig. 1 used ambient air compressed to 446 kPaa (65 psia) and dried to −45°C (−50°F) dew point. Wall static pressure taps, Fig. 1, were 0.8-mm (0.032-in.) diameter. A liquid manometer was used for small pressure differences in the inlet contraction and approach duct, and a Druck pressure transducer accurate to 1 in 3500 elsewhere. Mass flow rates accurate to ±1% were measured using a VDI nozzle of 40.47-mm (1.60-in.) diameter. Inlet flow Reynolds number was 8 × 10 based on pipe diameter.

Figure 2b compares computed and experimental wall-pressure distributions for case 1. The FEM predicts the extent of the region of adverse pressure gradient in the inlet section.

Meshes II-IV gave smooth velocity contours and streamlines for case 2, which contains a supersonic pocket, with a maximum wall Mach number of 1.13. The slower convergency of this case is consistent with the fact that the Galerkin criterion is basically an elliptic operator. The method was successfully applied to a nozzle with $R_* = 0.625$.

References

[1] Zienkiewicz, O. C., *The Finite Element Method in Engineering Science*, 2nd ed., McGraw Hill, London, 1971.

[2] Fletcher, C.A.J., "The Galerkin Method: An Introduction," *Numerical Simulation of Fluid Motion*, edited by J. Noye, North Holland, Amsterdam, 1978.

[3] Midgal, D., Klein, K., and Moretti, G., "Time-Dependent Calculations for Transonic Nozzle Flow," *AIAA Journal*, Vol. 7, Feb. 1969, pp. 372-374.

[4] Labujere, Th.E. and Van der Vooren, J., "Finite Element Calculations of Axisymmetric Subcritical Compressible Flow," NLR-TR 74162 U, 1974.

[5] Periaux, J., "3-D Analysis of Compressible Potential Flows with the Finite Element Method," *International Journal for Numerical Methods in Engineering*, Vol. 9, No. 4, 1975, pp. 775-832.

[6] Chan, S.T.K., Brashears, M. R., and Young, V.C.Y., "Finite Element Analysis of Transonic Flow by Method of Weighted Residuals," AIAA Paper 75-79, 1975.

[7] Shen, S. F. and Habashi, W., "Local Linearization of the Finite Element Method and its Application to Compressible Flow," *International Journal for Numerical Methods in Engineering*, Vol. 10, May-June 1976, pp. 565-577.

[8] Fletcher, C.A.J., "The Primitive Variables Finite Element Formulation for Inviscid Compressible Flow," *Journal of Computational Physics*, Vol. 33, Dec. 1979, pp. 301-312.

[9] Chattot, J. J., Guiu-Roux, J., and Laminie, J., "Finite Element Calculation of Steady State Transonic Flow in Nozzles using Primary Variables," *Lecture Notes in Physics*, Vol. 141, Springer-Verlag, Berlin, 1981, pp. 107-112..

[10] Back, L. H., Massier, P. F., and Gier, H. L., "Comparison of Measured and Predicted Flows through Conical Supersonic Nozzles with Emphasis on the Transonic Region," *AIAA Journal*, Vol. 3, Sept. 1965, pp. 1606-1614.

Frozen-Plasma Boundary-Layer Flows over Adiabatic Flat Plates

G. Ben-Dor,* Z. Rakib,† and O. Igra‡
Ben-Gurion University of the Negev, Beer Sheva, Israel

Introduction

THE nonlinear partial differential equations describing most boundary-layer problems are difficult to solve. Consequently, many investigators resort to using simplifying similarity transformations. In complex flows, where similarity solutions cannot be used, an exact solution for the general boundary-layer flow equations is not possible. Reviews of commonly used techniques for solution of boundary-layer problems can be found in Refs. 1-3.

One type of boundary-layer problem is the shock wave generated boundary layer. For strong shock waves, the shock-induced flow must be considered as real gas, i.e., plasma. The ionized gas can be in a nonequilibrium, equilibrium, or frozen

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*Senior Lecturer, Department of Mechanical Engineering.
†Ph.D. Student, Department of Mechanical Engineering.
‡Associate Professor, Department of Mechanical Engineering.

state, depending on the time given to the flow to relax to its appropriate equilibrium state.

In a recent work of Liu et al.,[4] the shock tube generated, flat-plate laminar boundary layer in argon was studied. In their study, the governing equations were solved using an implicit six-point, finite difference scheme for the three cases: i.e., nonequilibrium, equilibrium, and frozen flow. It is of interest to note that the numerical results obtained for the frozen flow case lie closer to the experimentally measured flow properties than the nonequilibrium values (see Figs. 13-15 and 18-20 in Ref. 4). Consequently, it is apparent that the frozen behavior could represent the shock tube generated flowfield over a flat plate quite well and should not be considered only as a pure academic exercise.

Motivated by this fact and by the known difficulties encountered in using the finite difference method for ionizing boundary layers, we proposed[6] a simple numerical approach which was originated by Töpfer[7] to solve the frozen case of a shock tube generated boundary-layer flow. This approach was also successfully used by us for the simpler case of a perfect gas.[5] The approach is based on the following "group property." The solution of Eqs. (1-3), i.e., $\phi(\beta)$, $H(\beta)$, and $\alpha(\beta)$ is invariant under the transformation: $\phi(\beta)=b\phi_0(a\beta)$, $H(\beta)=cH_0(a\beta)$, and $\alpha(\beta)=d\alpha_0(a\beta)$. The idea for using this type of transformation is due to Topfer, who solved Blasius' equation using the transformation $\phi(\eta)=\alpha\phi_0(\alpha\eta)$. Details concerning our solution can be found in Ref. 6.

In a previous Note[8] results for the case of an isothermal flat plate were presented. The present Note is aimed at presenting the results for the case of an adiabatic flat plate. This case should not be considered as only an academic exercise, since it might represent actual flows over models mounted in a shock tube. This is because the duration of such flows is too short for heat transfer to occur and, hence, can be assumed adiabatic.

Theoretical Background

For a supersonic flow over a flat plate, one can use the following assumptions: 1) two-dimensional steady flow; 2) laminar flow; 3) no (continuum) radiation losses; 4) no diffusion ($u=u_e$); 5) constant freestream conditions along the flow direction; 6) no electric or magnetic fields; 7) thermal equilibrium ($T=T_e$); 8) frozen flow, i.e., $\dot{\omega}_e=0$; 9) constant Le number; 10) constant Pr number; 11) $\rho\mu=H^n$ (this type of relation is based on the fact that for ideal gases $\rho\mu=(P/RT)C_1T^{0.76}=C_2T^{-0.24}$; hence for nonideal gases, a similar relation is assumed with an unknown power n); and 12) $I\gg(5/2)RT$ (i.e., $T\ll(2/5)\theta_I$, where T is the plasma temperature and θ_I is the characteristic ionization temperature. Thus, for argon ($\theta_I=182{,}850$ K), the following set of similar equations can be obtained[6]:

$$\phi\phi''+(H^n\phi'')'=0 \tag{1}$$

$$Pr\phi H'+(H^nH')'+(A/4)(H^n\phi'\phi'')'+BC(H^n\alpha')'=0 \tag{2}$$

$$Pr\phi\alpha'+Le(H^n\alpha')'=0 \tag{3}$$

where ϕ is the velocity potential function, H the normalized total enthalpy, and α the degree of ionization. Pr and Le are the Prandtl and Lewis numbers. A, B, and C are constants, depending on the freestream flow properties, and n is the exponential dependence of the density-viscosity product on the temperature as defined in assumption 11.

In the case of an adiabatic plate, the boundary conditions are $\phi(0)=0$, $\phi'(0)=0$, $H'(0)=(I\alpha_\infty/H_\infty)\alpha'(0)$ [see Appendixs in Ref. 6], $\alpha(0)=\alpha_w$, $\phi'(\infty)=2$, $H(\infty)=1$, and $\alpha(\infty)=1$. If the plate can be considered as relatively cold, then $\alpha_w=0$.

Since not all the boundary conditions are given at $\beta=0$ (i.e., some are given at $\beta=\infty$) the problem at hand is known as a two-point boundary value problem. One method for solving two-point boundary value problems is known as the shooting method.

The present results are based on a new approach for solving Eqs. (1-3). In this approach, a transformation by which the problem is reduced to a Cauchy problem was found (see Ref. 6).

Results and Discussion

Equations (1-3) enable us to solve the laminar boundary layer that develops over a flat plate when a compressible, singly ionized gas flows over it. Such flow can be induced by strong shock waves. For this case, the postshock equilibrium flow properties that are obtained behind the relaxation zone are the freestream properties for the boundary-layer flow.

In our analysis, we assumed that Pr and Le are constants. In general, they are functions of the plasma temperature and its degree of ionization. When the plasma is in a state of thermal equilibrium, its degree of ionization can be expressed in terms of temperature and pressure through the Saha equation. Since for boundary-layer problems with constant freestream pressure the pressure throughout the entire field can be considered as constant, it appears that for the problem at hand Pr and Le depend on the temperature only (see Fig. 13 in Ref. 3).

As shown in Ref. 8, Liu[3] has used in his frozen case study $\rho\mu=1$ (i.e., $n=0$), $Pr=1$, and $Le=1$. By doing so, he has artificially removed the coupling between Eqs. (1-3) and forced the system to reduce to a simple set. Our selected values for n, Le, and Pr ensure that the coupling between Eqs. (1-3) remains throughout the entire solution.

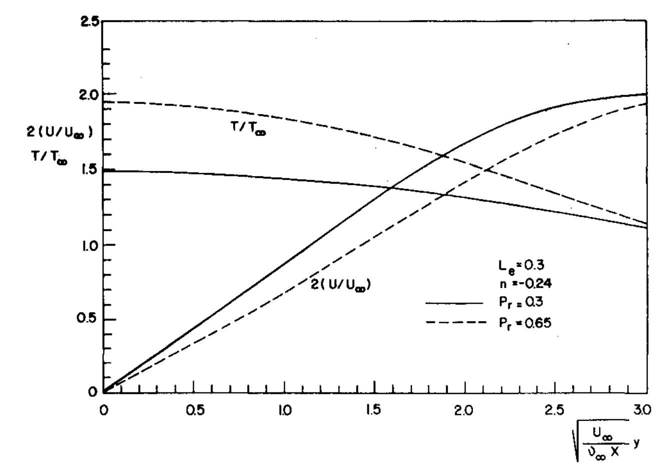

Fig. 1 Velocity and temperature dependence on *Pr*.

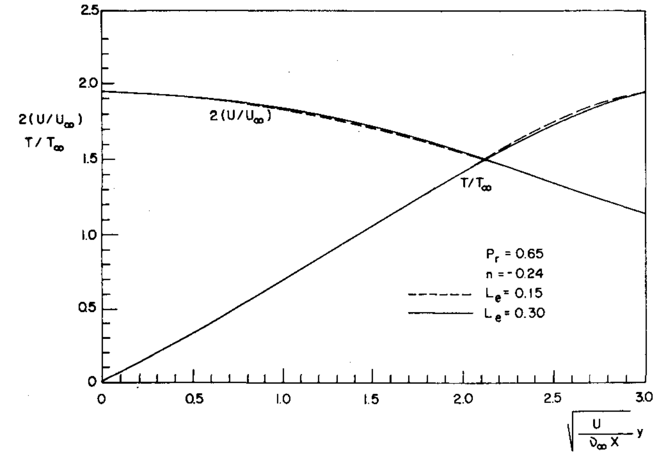

Fig. 2 Velocity and temperature dependence on *Le*.

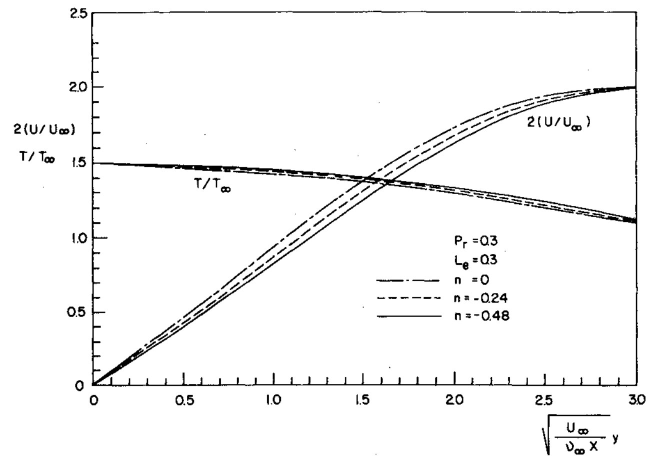

Fig. 3 Velocity and temperature dependence on n.

For an ideal gas, $n = -0.24$, in our solution we have used the following values: $n = 0$ (identical to Liu[3]), $n = -0.24$ (the value appropriate for an ideal gas), and $n = -0.48$. Using this wide range of n enabled us to study the dependence of the solution upon it. Similarly, we have chosen two different values for Le ($=0.3$ and 0.15) and Pr ($=0.3$ and 0.65). Thus, the dependence of the flowfield on Le and Pr was also studied.

The temperature and velocity dependence on Pr and Le are shown in Figs. 1 and 2, respectively. While a change from 0.3 to 0.65 in Pr indicates a noticeable change in the profiles (Fig. 1), a change from 0.15 to 0.3 in Le hardly affects the velocity and temperature profiles.

Figure 3 illustrates the dependence of the flowfield on n. It is clearly seen that the smaller n is (i.e., more negative), the longer it takes to reach the inviscid uniform flow properties (i.e., the boundary layer is thicker).

Conclusions

The boundary-layer equations for a partially ionized frozen flow over a flat plate has been solved using a new approach in which the problem at hand was reduced from a two-point boundary value problem to a Cauchy problem, thus offering a simple, stable, and relatively inexpensive solution technique.

The method was applied to a strong shock-induced argon flow over an adiabatic flat plate. Since the method requires constant values for the Prandtl number, the Lewis number, and the exponential dependence n of the density viscosity product $\rho\mu$ upon the temperature T, the dependence of the flow inside the boundary layer on Pr, Le, and n was investigated. It was found that while Pr and n strongly affect the obtained flowfield, the influence of Le is negligibly small.

As a closing remark, it is of interest to note again that the findings of Liu et al.[4] indicate that the frozen behavior represents the shock-induced flowfield over a flat plate quite well and, hence, the frozen solution should not be considered as purely a sterile exercise.

References

[1] Schlichting, H., *Boundary Layer Theory*, McGraw-Hill Book Co., New York 1960.

[2] Stewartson, K., "The Theory of Laminar Boundary Layers in Compressible Fluids," Oxford Mathematical Monographs, Oxford University Press, Oxford, England, 1964.

[3] Liu, W. S., "The Analysis of Shock Structure and Nonequilibrium Boundary-Layer Induced by a Normal Shock Wave in an Ionized Argon Flow," UTIAS Report No. 198, Univ. of Toronto, Toronto, Ontario, Canada, 1975.

[4] Liu, W. S., Whitten, B. T., and Glass, I. I., "Ionizing Argon Boundary Layer. Part 1. Quasi-Steady Flat-Plate Laminary Boundary-Layer Flows," *Journal of Fluid Mechanics*, Vol. 87, Pt. 4, 1978, pp. 609-640.

[5] Yaknot, A., Ben-Dor, G., Rakib, Z., and Igra, O., "A New Approach to the Solution of the Boundary Layer Equations of an Ideal Compressible Flow over a Flat Plate," *Aeronautical Journal*, Paper No. 845, Jan. 1981, pp. 34-35.

[6] Ben-Dor, G., Rakib, Z., and Igra, O., "The Boundary Layer of a Compressible Singly Ionized Frozen Flow over a Flat Plate-A New Method of Solution," Collection of Papers of the 25th Israel Annual Conference on Aviation and Astronautics, 1983.

[7] Töpfer, C., "Bemerkungen zu dem Aufsatz von H. Blasius," *Zeitschr. f. Math. a Phys.*, Vol 60, 1912, pp. 397-398.

[8] Ben-Dor, G. Rakib, Z., and Igra, O., "Frozen-Plasma Boundary-Layer Flows over Isothermal Flat Plates-Parametric Study," *AIAA Journal*, Vol. 22, Feb. 1984, pp. 299-301.

Shock Shape over a Sphere Cone in Hypersonic High Enthalpy Flow

S.L. Gai,* R.J. Sandeman,† P. Lyons,‡ and D. Kilpin§

The Australian National University, Canberra, Australia

Introduction

THE flowfield of a blunt-nosed body in hypersonic/hypervelocity flight has many complex features. These include subsonic flow in the stagnation region, flow rotationality, and, in addition, nonequilibrium phenomena such as dissociation and recombination. Under these circumstances, the mechanisms which influence pressures and heat-transfer rates on the surface of the body are the result of complex interaction of compression and expansion waves which originate due to body shape, reflection of waves from shock surface and sonic line, and wave reflections from the vorticity layer near the surface due to nonequilibrium flow.

As shown by Traugott,[1] this complex interaction is due, in part, to the transition that the flow has to make from the nose to the afterbody. In particular, at hypersonic Mach numbers, depending on whether the afterbody is slender (e.g., a hemisphere cylinder) or nonslender (e.g., blunted cone at an incidence), the flow may undergo either underexpansion or overexpansion. This would then cause marked deviations from thermodynamic equilibrium as theorized by Bloom and Steiger,[2] and confirmed by present results.

Flow Features

The overexpansion in the vicinity of transition leads to some pressure rise immediately following the expansion which is due to two reasons. Firstly, as a result of expansion and consequent thinning of the boundary layer in the shoulder region, the streamlines curve inward near the surface. This is evidenced by the interferometric observations of Giese and Bergdolt[3] on the flow over a truncated cone. Second, the curvature of streamlines gives rise to expansion waves which reflect back from the sonic line as well as from the bow shock as compression waves, which will then reinforce the afterbody (cone) shock thus giving rise to an inflection in the shock

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*Research Fellow, Department of Physics; presently, Dept. of Mechanical Engineering, University of New South Wales, Royal Military College, Duntroon, Australia. Member AIAA.

†Reader, Department of Physics.

‡Research Assistant, Department of Physics.

§Visiting Fellow, Department of Physics.